

1. Which of these ordinary differential equations is linear in the variables $x = [x_1, x_2]^T$ and $u \in \mathbb{R}$? (there are two)

(a) $\dot{x} = \begin{bmatrix} .2 & .4 \\ .5 & -.1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$

(b) $\dot{x} = \begin{bmatrix} x_1 & -x_2 \\ 3x_2 & -x_1 \end{bmatrix} x + \begin{bmatrix} u \\ 2 \end{bmatrix} u$

(c) $\dot{x} = \begin{bmatrix} u & -u \\ 3u & -u \end{bmatrix} x + \begin{bmatrix} x \\ 2 \end{bmatrix} u$

(d) $\dot{x} = \begin{bmatrix} \sin t & \cos t \\ t^2 & \sqrt{t} \end{bmatrix} x + \begin{bmatrix} e^t \\ 1 \end{bmatrix} u$

(e) $\dot{x} = \begin{bmatrix} x_1 & \cos x_2 \\ t^2 & u \end{bmatrix} x + \begin{bmatrix} 1 \\ x_2 \end{bmatrix} u$

2. Which of these are accurate descriptions of a first-order Taylor series of a function $f(x)$ at a point \bar{x} where $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ (there are two)

(a) $f(\bar{x}) \approx f(x) + \left[\frac{\partial f}{\partial x} \Big|_x \right] (x - \bar{x})$

(b) $f(\bar{x} + \Delta x) \approx f(\bar{x}) + \left[\frac{\partial f}{\partial x} \Big|_{\bar{x}} \right] \Delta x$

(c) $f(x) \approx f(\bar{x}) + \left[\frac{\partial f}{\partial x} \Big|_{\bar{x}} \right] (x - \bar{x})$

(d) $f(\bar{x} + \Delta x) \approx f(\bar{x}) + \left[\frac{\partial f}{\partial x} \Big|_{\Delta x} \right] \Delta x$

3. If we have a continuous time dynamical system described by $\dot{x} = Ax$, which of the following statements about the eigenvalues of A tells us that the system is stable?

- (a) the eigenvalues of A all have negative real parts
- (b) the eigenvalues of A are all real
- (c) the eigenvalues of A are all imaginary
- (d) the eigenvalues of A all have a modulus less than 1

4. If we have a discrete time dynamical system described by $x_{k+1} = Ax_k$, which of the following statements about the eigenvalues of A tells us that the system is stable?

- (a) the eigenvalues of A all have negative real parts
- (b) the eigenvalues of A are all real
- (c) the eigenvalues of A are all imaginary
- (d) the eigenvalues of A all have a modulus less than 1