1. Given $f(x) = 0.5x_1^2 + \sin x_2 + e^{x_3}$ where $x \in \mathbb{R}^3$, which of the following is $\partial f/\partial x$? (hint: $f : \mathbb{R}^3 \to \mathbb{R}$, think about the dimensions)

   (a) $\frac{\partial f}{\partial x} = \begin{bmatrix} x_1 \\ \cos x_2 \\ e^{x_3} \end{bmatrix} \in \mathbb{R}^{3\times 1}$

   (b) $\frac{\partial f}{\partial x} = \begin{bmatrix} x_1 & \cos x_2 & e^{x_3} \end{bmatrix} \in \mathbb{R}^{1\times 3}$

2. Given $f(x) = 0.5x_1^2 + \sin x_2 + e^{x_3}$ where $x \in \mathbb{R}^3$, which of the following is $\nabla_x f(x)$? (hint: $f : \mathbb{R}^3 \to \mathbb{R}$, think about the dimensions)

   (a) $\nabla_x f(x) = \begin{bmatrix} x_1 \\ \cos x_2 \\ e^{x_3} \end{bmatrix} \in \mathbb{R}^{3\times 1}$

   (b) $\nabla_x f(x) = \begin{bmatrix} x_1 & \cos x_2 & e^{x_3} \end{bmatrix} \in \mathbb{R}^{1\times 3}$

3. Given $f(x) = 0.5x_1^2 + \sin x_2 + e^{x_3}$ where $x \in \mathbb{R}^3$, is $\frac{\partial}{\partial x}\left(\nabla_x f(x)\right) = \nabla_x^2 f(x)$?

   (a) yes

   (b) no

4. Which one of the following pairs of vectors $x \in \mathbb{R}^3$ and $y \in \mathbb{R}^3$ **do not** exhibit complementarity?

   (a) $x = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

   (b) $x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

   (c) $x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

5. If we have the following optimization problem:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & c(x) = 0 \end{aligned} \tag{1}$$

   with a dual variable $\lambda$ associated with the constraint $c(x) = 0$ and a Lagrangian $\mathcal{L}(x, \lambda) = f(x) + \lambda^T c(x)$, we have the following KKT conditions:

$$\nabla_x \mathcal{L}(x, \lambda) = \nabla_x f(x) + \left[\frac{\partial c}{\partial x}\right]^T \lambda = 0, \tag{2}$$

$$c(x) = 0. \tag{3}$$

   Which one of the following linear systems computes the "full" Newton step, (the other being the Gauss-Newton step)?

   (a) $\begin{bmatrix} \nabla_x^2 f(x) & \left[\frac{\partial c}{\partial x}\right]^T \\ \frac{\partial c}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L}(x, \lambda) \\ -c(x) \end{bmatrix}$

   (b) $\begin{bmatrix} \nabla_x^2 \mathcal{L}(x, \lambda) & \left[\frac{\partial c}{\partial x}\right]^T \\ \frac{\partial c}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L}(x, \lambda) \\ -c(x) \end{bmatrix}$