1. We have the following optimization problem:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & c(x) \leq 0 \end{aligned} \tag{1}$$

with a dual variable λ associated with the inequality constraint $c(x) \leq 0$ and a Lagrangian $\mathcal{L}(x, \lambda) = f(x) + \lambda^T c(x)$. Which of the following is **not** one of the KKT conditions?

   (a) $\nabla_x f(x) + \left[\frac{\partial c}{\partial x}\right]^T \lambda = 0$
   (b) $c(x) \leq 0$
   (c) $\lambda \geq 0$
   (d) $c(x)^T \lambda > 0$
   (e) $\lambda \odot c(x) = 0$ ($\odot$ is element-wise multiplication)

2. If $a \odot b = 0$, does $a^T b = 0$? ($\odot$ is element-wise multiplication)

   (a) yes
   (b) no

3. If we have the following optimization problem:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & c(x) = 0 \end{aligned} \tag{2}$$

with the following KKT conditions:

$$\nabla_x \mathcal{L}(x, \lambda) = \nabla_x f(x) + \left[\frac{\partial c}{\partial x}\right]^T \lambda = 0, \tag{3}$$

$$c(x) = 0. \tag{4}$$

which of the following is the correct way to regularize the Gauss-Newton step computation with a regularizer $\beta > 0$? (The regularizer is shown in red.)

   (a) $\begin{bmatrix} \nabla_x^2 f(x) + \beta I & \left[\frac{\partial c}{\partial x}\right]^T \\ \frac{\partial c}{\partial x} & +\beta I \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L}(x, \lambda) \\ -c(x) \end{bmatrix}$

   (b) $\begin{bmatrix} \nabla_x^2 f(x) + \beta I & \left[\frac{\partial c}{\partial x}\right]^T \\ \frac{\partial c}{\partial x} & -\beta I \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L}(x, \lambda) \\ -c(x) \end{bmatrix}$

4. We have the following optimization problem:

$$\underset{a,b}{\text{minimize}} \quad (a-4)^2 + (2b+3)^2 - 3a + 2b + 3, \tag{5}$$

and we want to solve it in a linear system, where $Ax = b$ with $x = [a, b]^T$. What are A and b? To solve this problem, put it in a standard quadratic form and solve for when the gradient equals 0.

   (a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}, \quad b = \begin{bmatrix} -11 \\ 8 \end{bmatrix}$
   (d) $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ -14 \end{bmatrix}$