1. Is the matrix A^TA positive semi-definite? (hint, for A^TA to be positive semi-definite, $x^TA^TAx \geq 0$ for all x.
   (a) yes
   (b) no

2. Is the following linear system controllable?

$$x_{k+1} = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ .1 \end{bmatrix} u_k \tag{1}$$

   (a) yes
   (b) no

3. Below is a finite-horizon LQR problem,

$$\min_{x_{1:N}, u_{1:N-1}} \quad \sum_{i=1}^{N-1} \left[\frac{1}{2} x_i^T Q x_i + \frac{1}{2} u_i^T R u_i \right] + \frac{1}{2} x_N^T Q_f x_N \tag{2}$$

$$\text{st} \quad x_1 = x_{\text{IC}}, \tag{3}$$

$$x_{i+1} = Ax_i + Bu_i \quad \text{for } i = 1, 2, \ldots, N-1, \tag{4}$$

   where $Q \succeq 0$, $R \succ 0$, and $[A, B]$ is controllable. We can solve this problem with either convex optimization that solves for (x, u) directly, or a Ricatti recursion that solves for an optimal policy $u_i = -K_i x_i$. Will these two solutions be equivalent?

   (a) yes
   (b) no

4. If we modify the finite-horizon LQR problem to incorporate bound constraints on the controls, we get the following problem:

$$\min_{x_{1:N}, u_{1:N-1}} \quad \sum_{i=1}^{N-1} \left[\frac{1}{2} x_i^T Q x_i + \frac{1}{2} u_i^T R u_i \right] + \frac{1}{2} x_N^T Q_f x_N \tag{5}$$

$$\text{st} \quad x_1 = x_{\text{IC}}, \tag{6}$$

$$x_{i+1} = Ax_i + Bu_i \quad \text{for } i = 1, 2, \ldots, N-1, \tag{7}$$

$$u_{min} \leq u_i \leq u_{max} \quad \text{for } i = 1, 2, \ldots, N-1. \tag{8}$$

   If we take the Ricatti solution ($K_{1:N-1}$) from the original finite-horizon LQR problem, and apply the following policy:

$$u_i = \max(u_{min}, \min(-K_i x_i, u_{max})). \tag{9}$$

   Will we recover a solution to the new optimization problem? You should think about the theory as well as implementing it yourself and trying. First figure out what $\max(a, \min(u, b))$ does when $a < b$, making sure to use max.() and min.() if you're doing this in Julia.

   (a) yes
   (b) no

5. Which of the following cost functions are convex in $x \in \mathbb{R}^N$? ($Q \succ 0$)

   (a) $c^T x$
   (b) $-c^T x$
   (c) $\sum_i x_i$
   (d) $\|x\|_1$
   (e) $\|x\|_2$
   (f) $\|x\|_2^2$
   (g) $\|Ax - b\|_1$
   (h) $x^T Q x$
   (i) $(Ax - b)^T Q (Ax - b)$

6. Which of the following constraints are convex in $x \in \mathbb{R}^N$? ($Q \succ 0$)

   (a) $Ax = b$
   (b) $Ax \leq b$
   (c) $\|x\|_2 \geq 3$
   (d) $\|Ax - b\|_1 \leq 3$
   (e) $\|Ax - b\|_2 \leq 3$
   (f) $\|Ax - b\|_2 = 3$
   (g) $\|x\|_2^2 \leq 3$
   (h) $x^T Q x \leq 3$
   (i) $x^T Q x = 3$
   (j) $x^T Q x \geq 3$
   (k) $x^T Q x + q^T x \leq 3$